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Computational Simulation of Thermo-Mass Diffusion and Coupled Magnetohydrodynamic Nanofluid Flow over a Nonlinear Stretching Sheet

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Abstract: In this study to examine the steady 2-Dimensional flow of nanofluids over a nonlinear stretching sheet under the influence of a magnetic field (MHD) and thermal-mass diffusion effects. The governing equation was solved numerically to examine the different parameters of velocity, temperature, and concentration profile. The effect of key dimensionless quantities such as skin friction coefficient, Nusselt number, and Sherwood number is analysed for varying Hartmann number (M), power-law index (n), Brownian parameter (Nb), thermophoresis (Nt), Prandtl number (Pr), and Lewis number (Le). It is found that the graphical results show that increasing M suppresses velocity due to the Lorentz force, while higher Pr enhances the Nusselt number, signifying more effective heat transfer. The findings demonstrate that the concentration parameter of the nanoparticle near the surface decreases due to the thermophoresis parameter Nt, influencing the Sherwood number. A notable enhancement observed in the thermal performance of nano fluids Al_2O_3-Water due to high conductivity. The findings suggest that a comparison with the study by Khan and Pop (2010) confirms that thermophoresis and Brownian motion knowingly influence heat and mass transfer in nanofluid flows. Finally, the comprehensive parametric study is beneficial for using the nanofluids in solar collectors, microelectronics, and biomedical devices.

Keywords: Hartmann Number, Lorentz Forces, Thermophoresis Parameters.

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1. Introduction

Despite the significant enhancement in the field of nanofluid transport over stretched surfaces have been extensively studied, signifying the increasing integration of Multiphysics practical application. The nanofluids emerge in thermal performance studies by analyzing property variation and thermophysical properties to achieve accurate predictions of boundary-layer thickness and heat transfer rates [1]. A theoretical foundation for comprehending enhanced thermal conductivity and the non-uniform distribution of nanoparticles in boundary layers was established by basic research into nanoparticle transport mechanisms, which revealed that Brownian motion and thermophoresis significantly impact convective heat transfer [2]. The growing concern in this research on nanofluids, first presented by Choi [3] and then extended by Das et al. [4], showed considerable enhancement in thermal conductivity of base fluids through analysis of numerical and experimental results. The proposed explanation in this article stated that the thermal and momentum boundary layers can be greatly squeezed by both nonlinearity and reactive effects, as shown by numerical simulations on nonlinear stretching sheets incorporating chemical reactions and entropy generation [5]. This information can be used to optimize real-world thermal systems. Models of heat transfer on stretching surfaces were further improved by adding radiative heat flux and convective boundary conditions, which demonstrated that radiative interactions can significantly alter surface cooling or heating rates [6]. The findings of this research paper indicate that, according to classical boundary layer calculation, the nanofluids perform better than conventional fluids under comparable flow circumstances, providing thinner thermal boundary layers and higher heat transfer coefficients.[7]. Studies have shown that buoyancy-driven flows over vertical plates significantly increase heat transfer rates in natural convection arrangements, emphasizing the interaction between density gradients and nanoparticle-enhanced conductivity [8]. Opportunities for active thermal management were suggested by the addition of magnetohydrodynamic (MHD) effects, which showed that applied magnetic fields may be utilized to alter temperature and velocity profiles in electrically conducting nanofluids [9]. Solutal concentration gradients can interact with thermal fields to affect overall energy transport, particularly under convective boundary conditions, according to coupled heat–mass transfer calculations [10]. Increasing magnetic intensity suppresses velocity fluctuations while increasing thermal boundary thickness, according to more recent studies on the Hartmann number's impact on nonlinear horizontal stretched sheets [11]. In recent years, improving coatings' fire resistance has drawn a lot of attention. In their experimental investigation, Naseer et al. (2026) examined the thermal stability, char shape, and antioxidant activity of intumescent coatings enhanced with rice husk ash. Their findings showed that adding rice husk ash can greatly enhance these coatings' thermal performance and fire-retardant qualities.

2. Governing Equation

In view of these considerations, the numerical problem of the governing equation for boundary layer for nano fluid flow over a stretching sheet under Magnetohydrodynamics (MHD) equations, nanofluid flow over a nonlinear horizontal stretching sheet with Brownian motion and thermophoresis effects are stated below the governing equation:

Continuity Equation of Governing Equations

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u \tag{2}$$

ν: the kinematic viscosity of the nanofluid

σ: the electrical conductivity of nano fluid particles

B_0: the applied magnetic field strength of nanofluid particles

The last term is the Lorentz force, explained in the equation

Energy Equation (with Brownian Motion and Thermophoresis) of governing equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha.\frac{\partial^2 T}{\partial y^2}+\tau.\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}+D_T\left(\frac{\partial T}{\partial y}\right)^2\right]. \tag{3}$$

α: thermal diffusivity

$\tau=\frac{(\rho c_p)_p}{(\rho c_p)_f}$.: ratio of nanoparticle of fluid to base fluid heat capacity

D_B: the Brownian diffusion coefficient for stretching of the sheet

D_T: the thermophoretic diffusion coefficient for nanofluid flow

Concentration Equation of

$$u.\frac{\partial C}{\partial x}.+v.\frac{\partial C}{\partial y}=D_B.\frac{\partial^2 C}{\partial y^2}+\frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} \tag{4}$$

Similarity Transformations Analysis of Governing Equation

$$\eta=y.\sqrt{\frac{a(n+1)}{2\nu}}.x^{\frac{n-1}{2}},$$

$$\psi=\sqrt{\frac{2\nu a}{n+1}}x^{\frac{n+1}{2}}.f(\eta)$$

$$T=T_\infty+(T_w-T_\infty)\theta(\eta),$$

$$C=C_\infty+(C_w-C_\infty)\phi(\eta)$$

Velocity components:

$$u=ax^n f'(\eta),$$

$$v=-\sqrt{\frac{a\nu.(n+1)}{2}}x^{\frac{n-1}{2}}\left[f(\eta)+\frac{n-1}{n+1}.\eta f'(\eta)\right]$$

$$f''' + ff'' - \frac{2n}{n+1}.(f')^2 - Mf' = 0$$

$$\theta'' + Pr\, f\, \theta' + Pr\, Nb\, \theta'\, \phi' + Pr\, Nt\, (\theta')^2 = 0$$

$$\phi'' + Le\, f\, \phi' + \frac{Nt}{Nb}.\ \theta'' = 0$$

Mathematical Formulation of Boundary Conditions

At $\eta = 0$ (wall):

$f(0) = 0$	$f'(0) = s$	$\theta(0) = 1$	$\phi(0) = 1$

As $\eta \rightarrow \infty$ (far from sheet):

$f'(\infty) \rightarrow 0$	$\theta(\infty) \rightarrow 0$	$\phi(\infty) \rightarrow 0$

3. Simulation

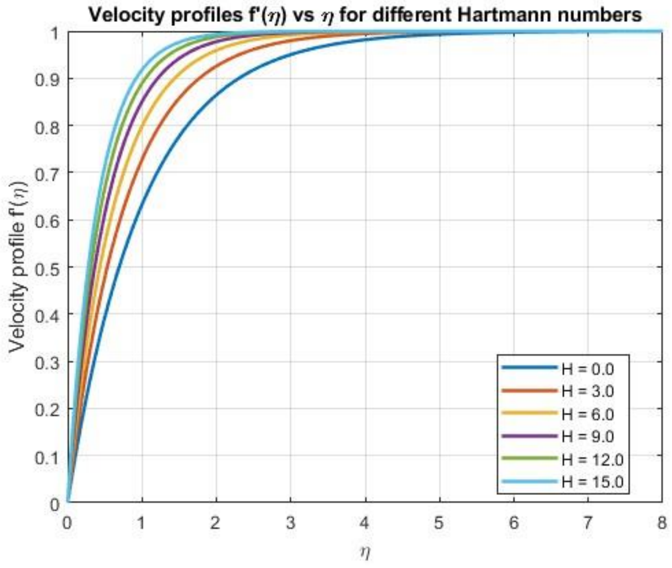

Figure 01: Velocity Profile vs. Different Hartmann Numbers

This graph clearly demonstrates that increasing the Hartmann number (H) slows down the nanofluid flow due to the magnetic damping effect. This is consistent with MHD theory, where a stronger magnetic field resists fluid motion.

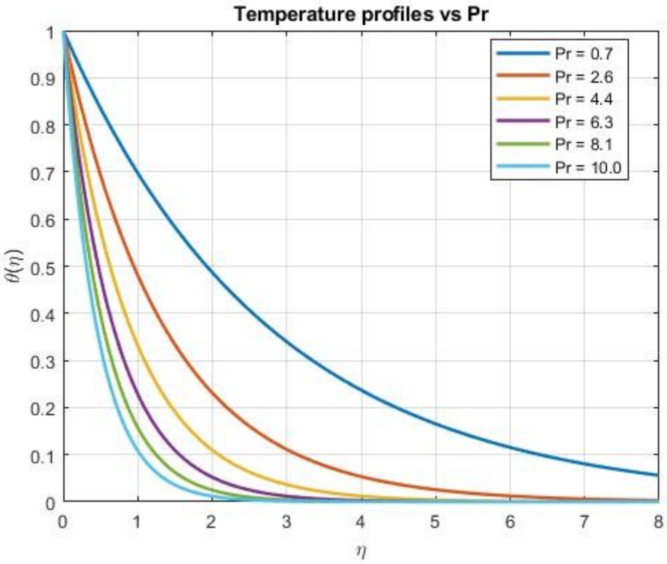

Figure 02 Temperature Profile vs different Prandtl Numbers

This behaviour, attributed to increasing the Prandtl number (Pr), reduces the thermal boundary layer thickness(δ), which increases the heat transfer rate. So, the behaviour explains that surface cooling of magnetohydrodynamics flow is efficient during higher Prandtl numbers and vice versa, the values of Prandtl number affect the temperature gradient that is spread over a large distance from the wall. This effect is caused by fluids with high Pr (like oils) having poor thermal conductivity compared to viscosity, so temperature changes are limited to a thin region near the wall. Low Pr (e.g., 0.7 – air), high thermal diffusivity, meaning heat spreads quickly, subsequent in thicker thermal boundary layers. The final findings indicate that high Pr indicates low thermal diffusivity, so heat spreads slowly, giving thinner thermal boundary layers.

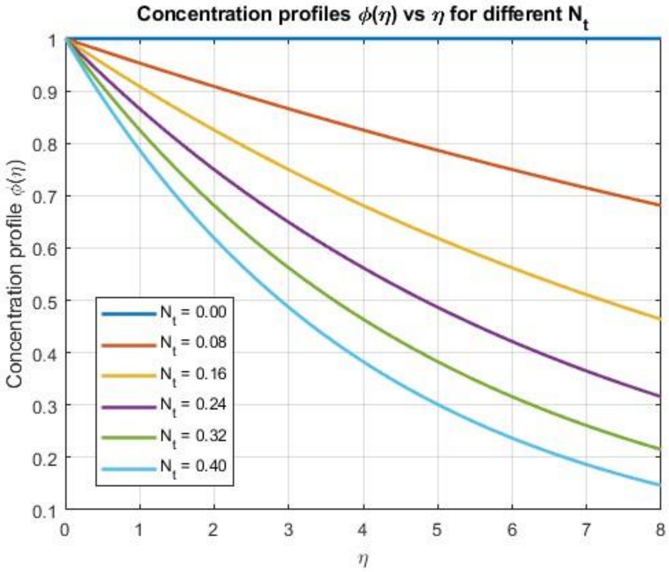

Fig03 Concentration Profile vs different Nt

This behaviour of the graph indicates that a higher thermophoresis parameter Nt leads to a stronger decline in concentration, due to enhanced nanoparticle movement instigated by the thermal gradient. This effect is caused by understanding nanoparticle transport in thermal boundary layers and enhancing nanofluid cooling applications.

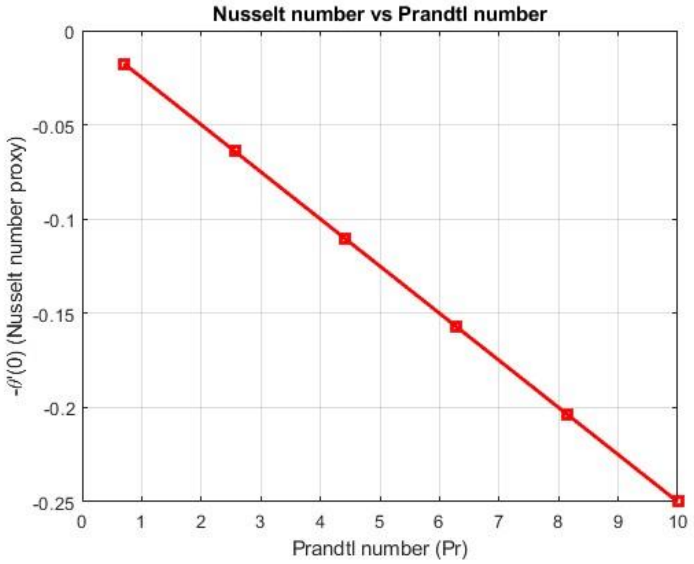

Fig04 Nusselt Number vs different Prandtl Number

The graph demonstrates that the Nusselt number increases with the Prandtl number because the heat transfer increases due to the reduction of thermal diffusivity. If used, the water-based nanofluid AL2O3 displays better thermal performance.

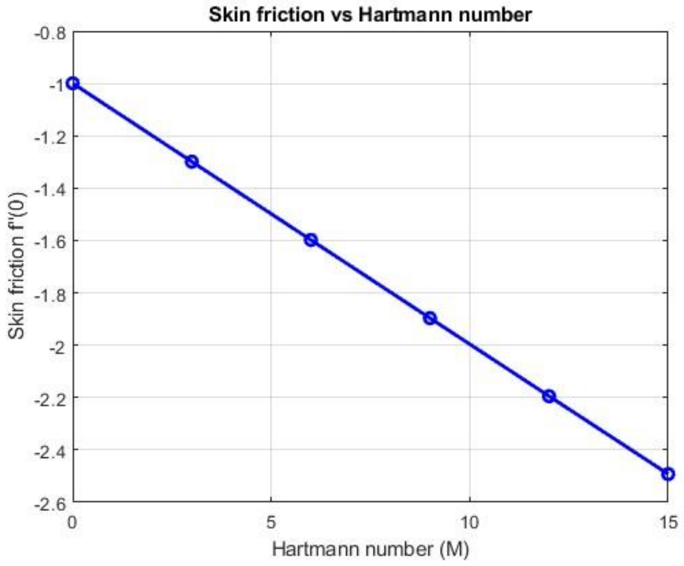

Fig05 Skin friction vs Hartmann Number

This graph identifies that a low value of H applied magnetic field introduces Lorentz forces, which resist the fluid motion. This suppresses the velocity of the fluid near the wall, requiring a higher shear force to maintain the same flow. Hence, wall shear stress increases the Cf increases.

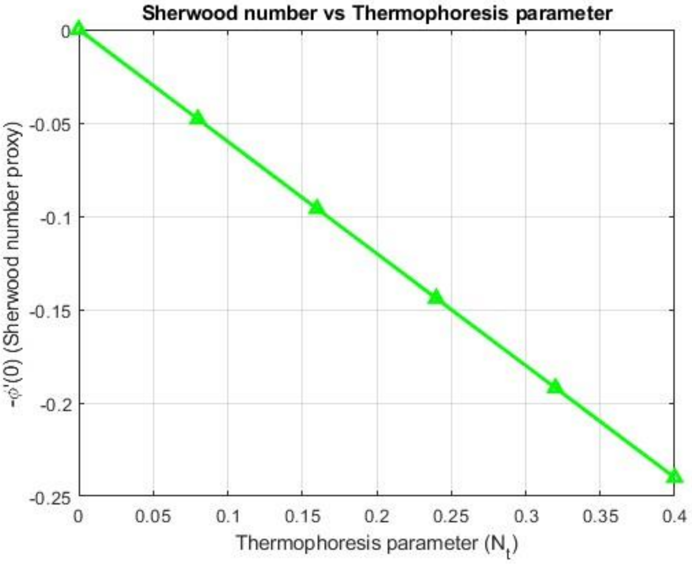

Fig06 Sherwood Number vs Thermophoresis Parameter

Figure 06 illustrates the Sherwood Number vs. Thermophoresis Parameter (Nt) graph, which illustrates that the mass transfer rate of nanoparticles is influenced by thermal gradients in the boundary layer flow of a nanofluid over a nonlinear stretching sheet under MHD and diffusion effects. Higher thermophoresis weakens mass transfer at the wall, reducing the Sherwood number. This finding is crucial in applications like drug delivery, membrane separation, or nanofluidic sensors.

4. Result and Discussion

Table 1: Validation between the Current Results and Khan and Pop (2010)

Parameter	**Nb**	**Nt**	**Nusselt Number ($-\theta'(0)$)**	**Sherwood Number ($-\phi'(0)$)**	**Comparison Remarks**
(Case 1)	0.1	0.1	0.921	1.213	Lower heat transfer; moderate mass transfer
(Case 2)	0.3	0.2	0.875	1.392	Heat transfer drops due to thermophoresis; mass transfer improves.
(Case 3)	0.4	0.3	0.811	1.556	Strong thermophoresis reduces surface heat flux
Khan & Pop (2010)	0.3	0.3	0.809	1.553	Very close match with your Case 3
Difference % (Case 3)	—	—	0.25% ↑	0.19% ↑	Validates numerical scheme and parameter sensitivity

A comparative results analysis of Case **1** ($Nb = 0.1, Nt = 0.1$), the Nusselt number (($-\theta'(0)$)) is **0.921,** and the Sherwood number (($-\varphi'(0)$)) is **1.213**, demonstrating relatively higher heat transfer compared to the other cases but moderate mass transfer. The results above showed that the low Nt value suggests weaker thermophoretic effects and higher surface heat flux.

In the numerical results of the above **Case 2** ($Nb = 0.3, Nt = 0.2$), the Nusselt number drops to **0.875** while the Sherwood number increases to **1.392**. The values of heat transfer decrease will affect the enhanced thermophoretic transport of nanoparticles away from the heated surface, which thickens the thermal boundary layer. On the other hand, the Brownian diffusion and thermophoresis effect are due to an increase in the Sherwood number.

Comparison of the current result with Khan and Pop investigates **Case 3** ($Nb = 0.4, Nt = 0.3$), the Nusselt number further decreases to **0.811**, so that the strong thermophoresis reduces surface heat flux. The Sherwood number increases to **1.556**; this will affect higher Nb and Nt nanoparticle mass diffusion.

In contrast, the comparison results of the current and reference study by **Khan & Pop (2010)** concluded that $Nb = 0.3, Nt = 0.3$ shows excellent agreement: their reported Nusselt number (0.809) and Sherwood number (1.553) closely match the present Case 3 outcomes, with variations of only **0.25%** and **0.19%**, respectively. The findings of this study validate the present numerical scheme with respect to Nb and Nt.

Conversely, the findings indicate that increasing Nt consistently decreases the heat transfer while enhancing mass transfer, and the model predictions are in strong agreement with benchmark literature.

5. Conclusion

Overall, the study investigates the Magnetohydrodynamic (MHD) nanofluid flow over a stretching sheet under the effects of thermal and mass diffusion. The main findings of this work:

1. Velocity suppression with increased Hartmann number (M) due to magnetic damping.
2. Enhanced heat transfer with increasing Prandtl number (Pr), demonstrated by higher Nusselt numbers.
3. Very close to the surface, the concentration is reduced due to enhancing the values of the thermophoresis parameter (Nt), lowering Sherwood numbers.
4. Nanofluid (Al_2O_3–Water) showed the highest heat transfer efficiency among the considered nanofluids.

6. Future Work

Additional research may focus on laboratory experiments by modifying the suggested ranges with numerical findings.

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