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Numerical Analysis of Hartmann Number Influence on Nonlinear Horizontal Stretched Sheet

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Abstract. This study examines numerically the problem of Hartmann number influence on two medium conduction-convection (conjugate) flow of nanofluids on a nonlinear horizontal stretched sheet. This model nonlinear partial differential equation solved by using Keller box method for nanofluid integrates the influence take on the buoyancy parameter, solute buoyancy, Brownian motion, thermophoresis parameter, Prandtl number and Lewis number found to have a strong effect on the system. The numerical result showed the velocity profile decreases during Prandtl number increases and temperature of Nano fluid increases under the influence of Hartmann number. The reduced Nusselt number when Prandtl number rises and reduced Sherwood number for large values of Lewis number. The Hartmann number influence and different parameters are presented through table and graph.

Keywords: Conduction/Convection (Conjugate) heat effects, Nano fluids, Nonlinear Horizontal stretched sheet, Hartman number Hx. Numerical solutions, Implicit finite difference scheme (Keller box method).

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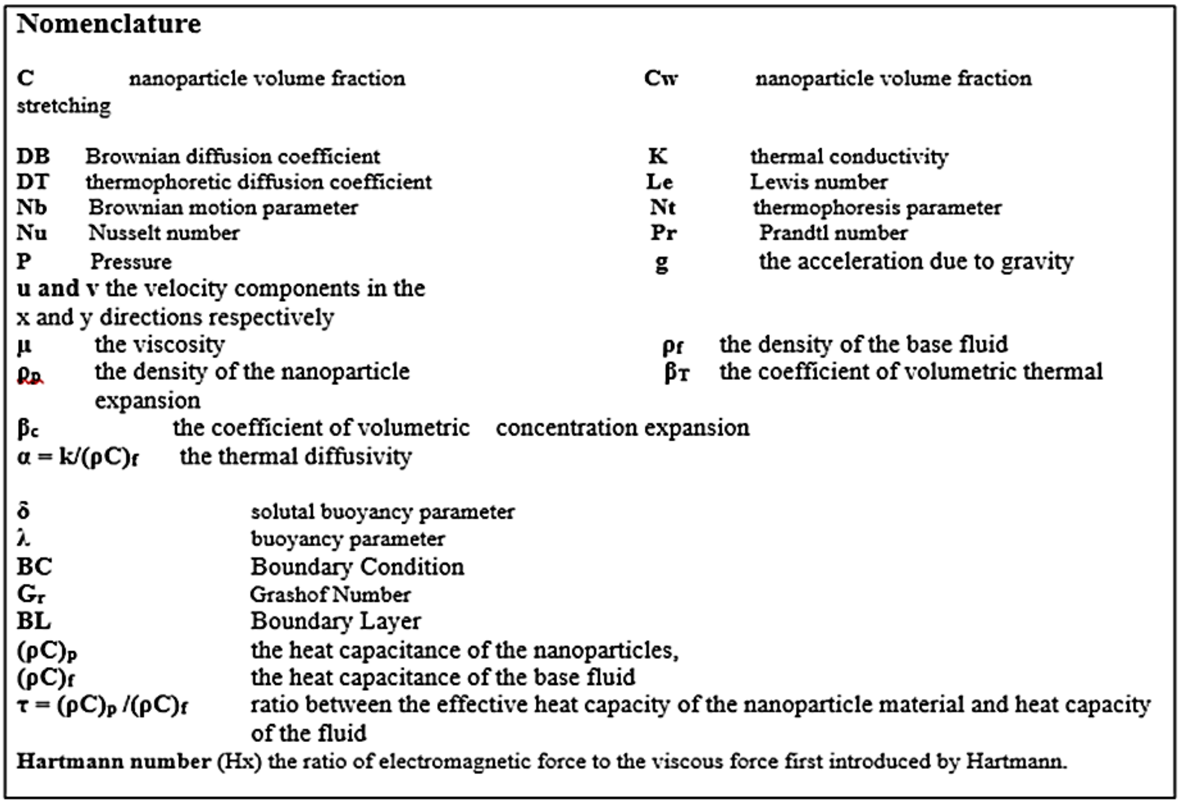
1. Introduction

The study of conjugate effects on stretching surfaces is important for many industrial applications like an extrusion molding, glass fiber, paper production industry hot rolling, and movement of biological fluids (Anwar et al., 2012). The inertia slips flow transfer of heat over a flat Stretching sheet in the presence of soret and dofour effect will directly affect heat transfer (Afify, 2009). The thermal diffusion effect of convective heat transfer over a Horizontal Stretched sheet considering injection or suction with thermal diffusion so the suction and injection have a significant impact on heat transfer (Aman and Ishak, 2010). This research paper on a vertical Surface effect of thermal diffusion identifies the surface heat flux. The base fluids poor heat transfer properties were observed as a great hurdle to the high solidity and value of heat exchangers (Daungthongsuk and Wongwisis, 2007). To increase the thermal conductivity, solid nanoparticles may be suspended into the base fluid. The first step is to get the base or conventional fluids having thermal conductivity hundreds of time lower than solid particles (Choi, 1995). They have inspected the impact of nanoparticles on characteristic convection limit layer stream past a vertical plate, utilizing a model in which Brownian movement and thermophoresis are represented. The writers have accepted the most straightforward conceivable limit conditions, in particular those in which both the temperature and the nanoparticle division are steady along the divider (Kuznetsov and Nield, 2010).



Nomenclature

C	nanoparticle volume fraction stretching	**Cw**	nanoparticle volume fraction
DB	Brownian diffusion coefficient	**K**	thermal conductivity
DT	thermophoretic diffusion coefficient	**Le**	Lewis number
Nb	Brownian motion parameter	**Nt**	thermophoresis parameter
Nu	Nusselt number	**Pr**	Prandtl number
P	Pressure	**g**	the acceleration due to gravity
u and v	the velocity components in the x and y directions respectively		
μ	the viscosity	**ρ_f**	the density of the base fluid
ρ_p	the density of the nanoparticle expansion	**β_T**	the coefficient of volumetric thermal
β_c	the coefficient of volumetric concentration expansion		
$\alpha = k/(\rho C)_f$	the thermal diffusivity		
δ	solutal buoyancy parameter		
λ	buoyancy parameter		
BC	Boundary Condition		
G_r	Grashof Number		
BL	Boundary Layer		
$(\rho C)_p$	the heat capacitance of the nanoparticles,		
$(\rho C)_f$	the heat capacitance of the base fluid		
$\tau = (\rho C)_p / (\rho C)_f$	ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid		

Hartmann number (Hx) the ratio of electromagnetic force to the viscous force first introduced by Hartmann.

In this work, the nano-fluid properties presented the stream over a stretching sheet. Nano fluid flow in a porous channel (well known as Cheng-Mincowcz problem) has been analyzed. Similarity method and porous series method laminar buoyant flow of bathing (Nield and Kuznetsov, 2010).

. On a moving plate analysis, the conjugate effect for different parameters configuration. Numerical study in a pour's medium analysis on the free convective Boundary Layer (Gdalevich and Fertman, 1977). In a numerical study of the Conjugate Heat Transfer in a vertical hollow cylinder heat transfer is calculated by free convection. Recently, Rani and Kim (Char et al., 1990). This research investigates that conjugate heat transfer depends on the open cavity in a free convection Boundary Layer with the high viscosity porosity surface Conjugate Heat transfer effect calculated by Darcy mixed convection problem. The temperature is dependent on the fluid viscosity now a day the conduction phenomenon resistance at the surface of the wall effect presents on the wall geometries. The stream flow and heat transfer induced, for example, in the air space double-pane window system varies basically from the peripheral natural convection in which the boundary layer was considered. (Pop and Na, 2000). In this research paper, the stagnation slip flow at stretching of the sheet surfaces was investigated by the homotropy analysis method (Abbas and Hayat, 2011).

2. Governing Equation

The solution of governing equations is presented here based on the implicit finite difference scheme. This two-dimensional assumption if apply the external pressure on a surface of a sheet in the x-direction is having a diluted Nano particle of the sheet in the x-direction is having a diluted Nano particle. The basic parameters and equations for nano fluids can be written in Cartesian coordinates x and y. Nano-fluid boundary layer flow on stretching surfaces has been studied for purpose of heat and mass transfer investigation.

The two-dimensional measurement through along the x-axis above the flow of Nano fluid and y ≥ 0. when applying the two equal but opposite forces along the x-axis then the sheet will be stretched. The stretched sheet surface temperature T and the Nano particle fraction C yield constant values T_w and C_w. The outer/ambient values are T_∞ and $C\infty$ are approaches to y infinity.

The Study flow of a Nano fluid towards a flat horizontal surface, under the effect of an external B_0. It is assumed that the flow is laminar whereas the fluid is incompressible and electrically conducting and applying the external magnetic effect. Further, the Cartesian coordinate system is the most appropriate for the present problem, with the *x*-axis being taken along the surface and the *y*-axis being assumed to be in the normal direction.

Under these considerations, the governing equations for the problem with boundary layer approximations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u.\frac{\partial u}{\partial x}+v.\frac{\partial u}{\partial y}= v.\frac{\partial^2 u}{\partial y^2}, \quad - \quad \frac{1}{\rho_f}\sigma_f\beta_0^2 u \quad +[(1+C_\infty)\rho_f\beta_T(T-T\infty)-(\rho_p-\rho_{p_\infty})\beta_c(C-C_\infty)]g$$
(2)

$$u.\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\tau\left[D_B\left(\frac{\partial C}{\partial y}\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] \quad (3)$$

The boundary conditions are

$$u=\lambda U_w,\quad v=v_w,\ T=T_w,\quad C=C_w,\ at\ y=0,$$
(4)

$$u\to 0,\ T\to T_\infty, C\to C_\infty,\quad as\quad y\to\infty \quad (5)$$

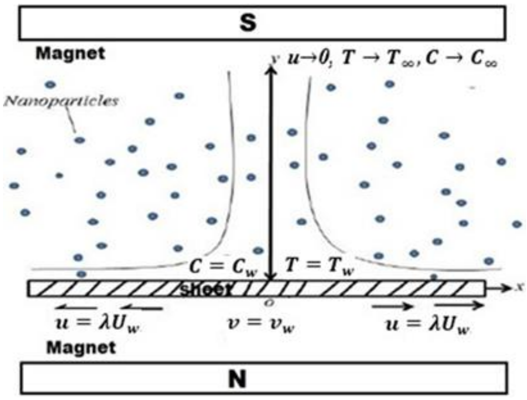


Fig. 1. Horizontal Stretched sheet with Magnetic Effect

Where the horizontal and normal velocity constituents, p are the pressure. The flat surface of the sheet is considered to be at the constant temperature T_w whereas the atmosphere temperature of the fluid far from the surface is T_∞.

Under these boundary conditions, we intended to solve the governing equations of the problem. To accomplish this, we introduce the following similarity transformations.

$$\eta=y\sqrt{\frac{a(n+1)}{2v}}x^{(n-1)/2}, u=ax^n f'(\eta) \quad (6)$$

$$v=-\sqrt{\frac{av(n+1)}{2}}x^{(n-1)/2}\left[f(\eta)+\frac{n-1}{n+1}\eta f'(\eta)\right] (7)$$

$$\theta(\eta)=(T-T_\infty)/(T_w-T_\infty),\quad \emptyset(\eta)=(C-C_\infty)/(C_w-C_\infty)$$

With:

$$v_w=-\sqrt{\frac{av(n+1)}{2}}x^{(n-1)/2}S$$

Where *S* is the constant suction/injection parameter.

The flow field has given above identically satisfies the continuity equation, which means that Eq. (7) is compatible with the continuity equation (1), and hence represents the possible fluid motion.

The fluid flow satisfies the continuity equation, which means that Eq. (7) is well-matched with the continuity equation, and the following represents the possible fluid motion. Substituting Eq. (7) into the mathematical model, we obtain:

$$f''' + ff'' - \frac{2n}{n+1}f'^2 + \frac{2}{n+1}(\lambda\theta - \delta\phi) = 0 \qquad \mathbf{(8)}$$

$$\frac{1}{Pr}\theta'' + f\theta' + Nb\theta'\emptyset' + Nt\theta'^2 = 0 \qquad \mathbf{(9)}$$

$$\emptyset'' + \frac{1}{2}Lef\emptyset' + \frac{Nt}{Nb}\theta'' = 0 \qquad \mathbf{(10)}$$

Whereas the boundary conditions reduce to

$$f(0) = S, \quad f'(0) = \lambda, \quad \theta(0) = 1, \ \emptyset(0) = 1 \qquad \mathbf{(11)}$$

$$f'(\eta) \to 0, \quad \theta(\eta) \to 0, \emptyset(\eta) \to 0 \ as\ \eta \to \infty$$

With $Pr = v/\alpha$ is the Pr no, and $Le = 2v/D_B$ is the Lewis no. Finally,

$$Nb = D_B\frac{\tau(C_w - C_\infty)}{v}. \quad Nt = D_T\frac{\tau(T_w - T_\infty)}{vT_\infty}$$

Are respectively, the Brownian motion parameter and thermophoresis parameter.

Here

$$\lambda = \frac{Gr}{Re_x^{\frac{3}{2}}}, \quad \delta = \frac{Gm}{Re_x^{\frac{3}{2}}}, \quad \mathrm{Pr} = \frac{v}{\alpha}, \quad Le = \frac{v}{D_B}, \quad v = \frac{\mu}{\rho_f}$$

$$Nb = \frac{\tau D_B(Cw - C\infty)}{v}, Nt = \frac{\tau D_T(Tw - T\infty)}{vT_\infty}, \quad Re_x = \frac{u_\infty(x)x}{v}$$

$$Gr = \frac{(1 - C_\infty)\left(\frac{\rho_{f\infty}}{\rho_f}\right)gn(T_w - T_\infty)}{v^2Re_x^{\frac{3}{2}}}$$

$$Gm = \frac{\left(\frac{\rho_f - \rho_{f\infty}}{\rho f}\right)gn_1(C_w - C_\infty)}{v^2Re_x^{\frac{3}{2}}}$$

Where f, θ and ϕ are stream function, temperature and concentration of nanoparticle.

The boundary condition transformed in to;

f = 0, f’ = 1, θ =1, ϕ = 1 at η = 0

$$f' \longrightarrow 0, \ \theta \longrightarrow 0, \ \phi \longrightarrow 0 \text{ when } \eta = \infty$$

The nonlinear ordinary differential equations 8, 9.and 10 use the following boundary conditions by implicating a finite difference scheme. The reduced the Nusselt number –θ (0) and reduced the Sherwood number – ϕ (0) under the influence of Hartmann number as defined as:

$$Nu_r = \frac{Nu}{\sqrt{m+\frac{1}{2}Rex}}, \quad Sh_r = \frac{Sh}{\sqrt{m+\frac{1}{2}Rex}}$$

3. Solution Procedure and Main Results

In this research, the governing Equations 8, 9 and 10 imposed boundary conditions Eq. 11 are solved for the numerical study by using the Keller box method.

1. We reduce the equations 8, 9. and 10 of the nonlinear system to a First-order System (FOS).

2. We mark the variance equation spending central difference scheme (Keller box method).

3. We practice the block tridiagonal elimination technique (BTET) to explain the linear system.

This solution procedure is extensively used which is earlier, easier to program, more effective and stretchier.

4. Result and Arguments

The physical amounts of our intrigued are the Liquid speed, stream astute speed, concentration Profile and temperature profile at the extending sheet dividers. The parameters of the issue are the Reynolds number Re, the attractive parameter Hx, the Prandtl number Pr, the Brownian movement Parameters Nb, thermophoresis Parameters Nt. and suction infusion Parameters. These parameters are all dimensionless bunches of fabric and stream properties, and/or geometric measurements of the space. The results of numerical for certain physical limitations are shown in tables and numerous figures.

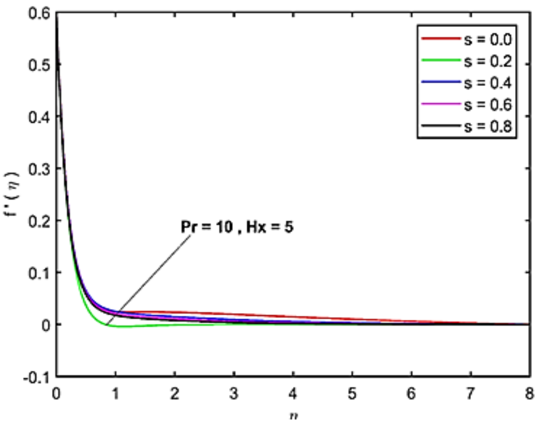


Fig. 1(a) Velocity Variation with different values of *s* and *Pr* number and Hartmann number *Hx=0*

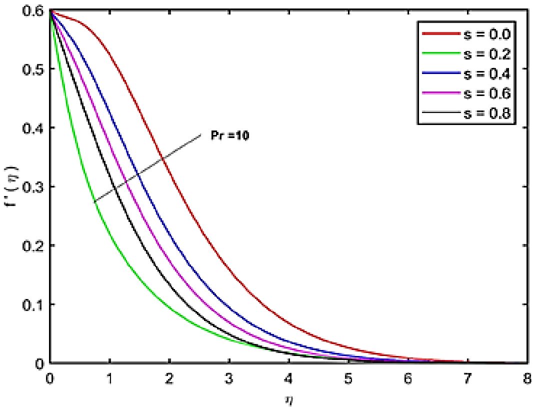


Fig. 1(b) Velocity Variation with different values of *s* and *Pr* number and Hartmann number *Hx=5*

The numerical result for some physical parameters of our attention according to this parameter λ=1.0, δ =1.0 Le=10 and Nb=Nt=0.1 without the effect of external magnetic effect f'(η) increases with increasing values of s and lower Pr number 0.71 fluid is being injected. However, the high thermal conductivity (K) and low Prandtl numbers (Pr) of liquid metals indicate that heat transfer by molecular thermal conduction is important not only in the near-wall layer. When the Pr number increases at point s=0.2 the velocity decreases because when the Prandtl number increases the liquid will be thicker. The outside magnetic field Hx effect on the normal velocity is presented in figure 1(b).it can be shown that with an increase of magnetic field parameter Hx=5.0 the fluid velocity decreases those forces which affect this normal velocity is called Lorentz forces. These forces affect the slowdown motion of the fluid.

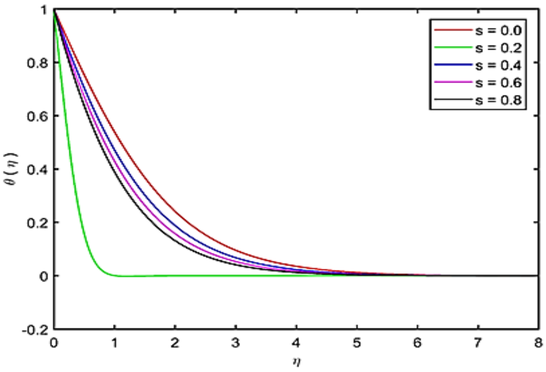


Fig. 2(a) Temperature Profile Variation with different values of *s* and *Pr* number and Hartmann number *Hx=0*

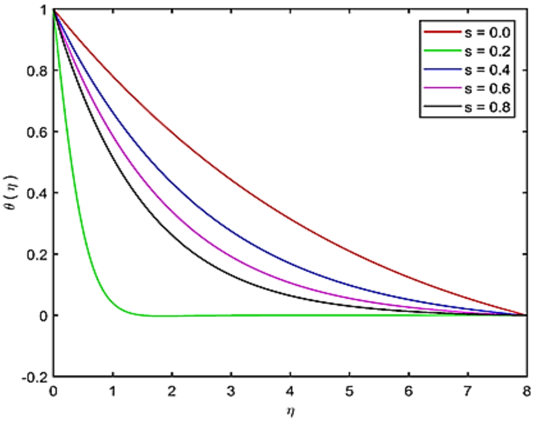


Fig. 2(b) Temperature Profile Variation with different values of *s* and *Pr* number and Hartmann number *Hx=5*

The numerical result for some physical parameter of our attention according to this parameter *λ=1.0, δ =1.0 Le=10 and Nb = Nt = 0.1* without the effect of external magnetic effect *θ(η)* The thermal properties of liquid metal be influenced by only temperature.

The minor transverse thermionic alteration in liquid metal flow is mainly due to the high thermal conductivity*(k)*, the effect of no isothermal conditions in heat transfer is not significant and, as a rule, is not careful during the *Hx* increases then the fluid temperature will be increased in figure 1 presents that when there increases the Pr number then Nano fluid will be thicker and during heating, it will be near the hot plate fewer values of *Pr* number are more effective.

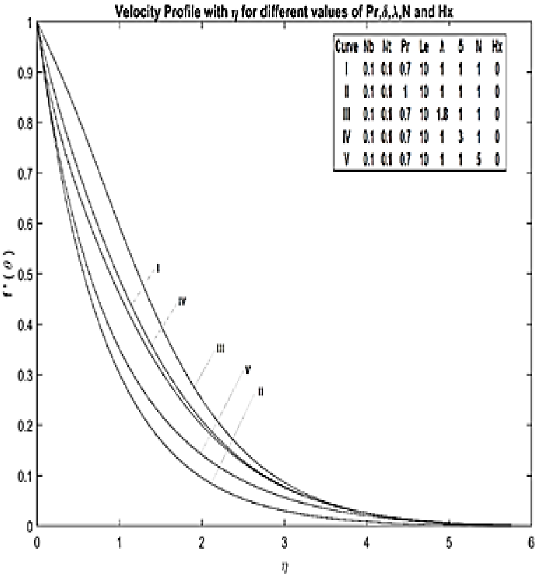


Fig. 3(a) Dissimilarity of *f'(η)* Profile along *η* for dissimilar Values of Prandtl, *λ* and *δ, N* and Hartmann number.

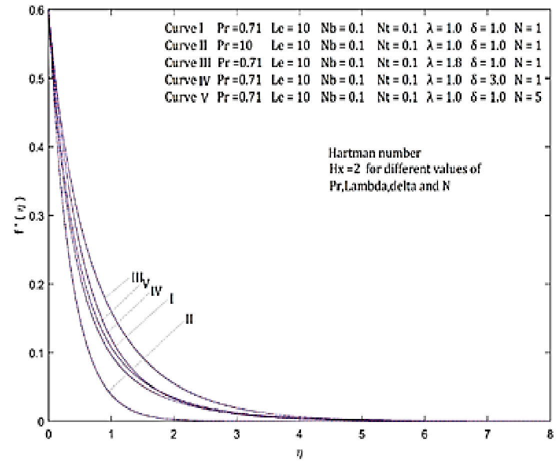


Fig. 3(b) Dissimilarity of f'(η) Profile along *η* for dissimilar Values of Prandtl, *λ* and *δ, N* and Hartmann number.

Variation of velocity on a stretching surface is present in two different figures with dissimilar values of Prandtl, *λ, δ, N* and *Hx*. In this fig. 3(a) present without Hartmann number effect at constant values of thermophoresis parameter *Nt* and Brownian motion *Nb* with dissimilar parameter *Pr*, buoyancy parameter, and solute buoyancy show that values of Brownian motion impact a large extent of fluid it results thickening the boundary layer during the small Prandtl number the boundary layer velocity increases when increases the values of *Pr* number decreases the velocity of the fluid. We observe Pr number increases fluid to move more viscous which leads to a decrease in the velocity. In figure 2 observe that curve I comparison with fig 1 show that the presence of Hartmann number the fluid velocity decreases curve travel below side near the hot stretching plate they present that magnetic effect on Nanoparticle movement decreases. Curve II presents that if the value of *Pr* number decreases then fluid will be more viscous velocity decreases this comparison shows present at the presence of Hartmann number effect fluid velocity will decrease smoothly. Curve III presents that comparison with fig 3(a) and 3(b) no wide effect on the flow because when the value of *λ* increases but at the presence of

Hartmann number velocity of the fluid will be the same. Comparison of Curve III with fig 3(a) and 3(b) presents when values of δ increase, but at the presence of *Hx* the fluid velocity will decrease. Curve *V* presents at the presence of Hartmann number there is a minor increase in velocity great difference occurs.

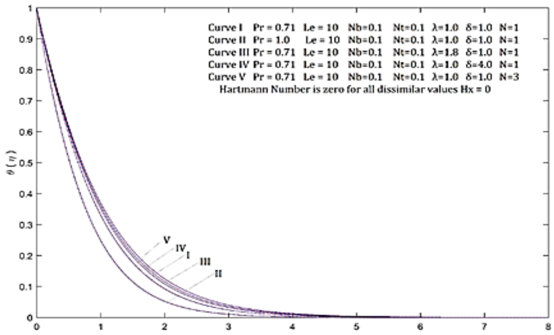


Fig. 4(a) ***Variation of Temperature Profile with η for dissimilar Values of Pr, λ and δ, and N***

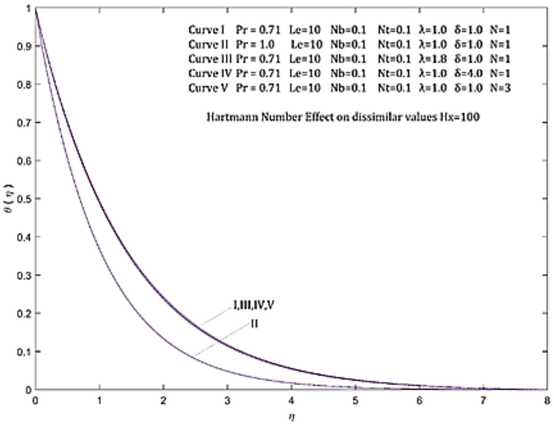


Fig. 4(b) ***Dissimilar Values of Pr, λ and δ, N and Hx for Variation of Temperature Profile***

The numerical result for some physical parameter of our attention according to dissimilar parameter without the effect of external magnetic Hx =0 effect θ(η) Figure 4(a)examine the different configuration of Pr,λ and δ on θ(η) profile. The analysis of the profile shows that when we increase values of Pr and λ the temperature profile decreases they increase when we increase the value of δ. The major reason is that when the Pr number increases the boundary layer of nano particle thick so suspended particle motion is affected. Figure 4(b) plotted to examine the no wide variation occur due to Hartmann number effect on the boundary layer *Pr* and *λ* result in a minor increase in the temperature profile while this profile rises when we apply much more value of Hartmann number so at the temperature profile no major effect of *Hx*.

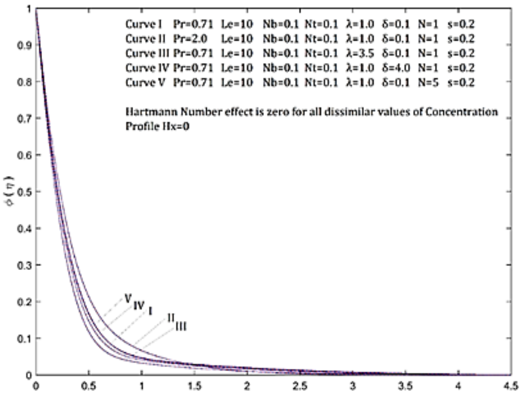


Fig. 5(a) ***Dissimilar Values of Variation of Concentration Profile***

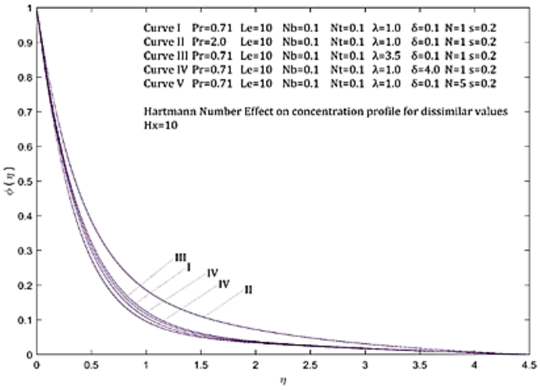


Fig. 5(b) ***Dissimilar Values of Variation of Concentration Profile with effect of Hartmann number Hx=10***

Figure 5 is ready to think about the impacts of disparate arrangement parameters on concentration profiles. It has appeared concentration profile the expanding values of λ basis a diminish within the mass exchange in spite of the fact that, the concentration profiles ϕ (η) increment for huge values of Pr and δ. In figure 5(b) analysis the Hartmann number effect curve travel away from the sheet. Hartmann number scatter the nanoparticle in fluid which effect transfer of heat maximum.

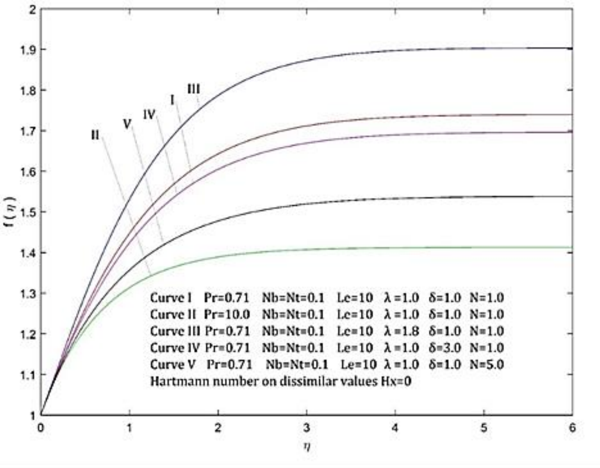


Fig. 6(a) ***Normal velocity with different Parameter's simulation without Hartman number***

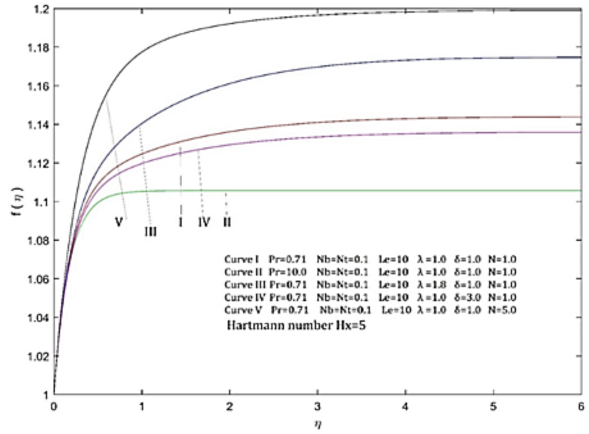


Fig. 6(b) ***Normal velocity with different Parameter's simulation with Hartman number***

i. The normal velocity at a larger value of the Brownian motion parameters (Nt) influences a huge presence of the fluid that generates the thickening of the thermal boundary layer (TBL).

ii. When we increase the values of the thermophoresis parameter (Nb) it results in the diffusion is piercing into the fluid and causing the TBL to be thicker.

iii. We take various values of the Brownian motion, thermophoresis parameter (Nt), Pr, Le,λ and δ are shown in figure 1 when there is an increase in the values of Pr result in a decrease a velocity.

iv. Only velocity increases when there is an increase in the value of λ because when we increase the values of Pr number then the fluid is more viscous which declines the velocity of Nano fluids.

v. Velocity declines when increasing the values of δ and N.

vi. When Hartman number influences fig 6(b) the pours stretching sheet when we increase the values of δ and N then due to magnetic effect velocity of the Nano fluid will increase.

The numerical configuration shows that in figure 7(a,b) present that taking the constant values of Pr number and Le number the velocity f'(η) increases when we increase the values of λ. The effect of Hartmann number in figure 7(b) the velocity decreases because Lorentz forces attract toward the Nano particle the heat transfer increases.

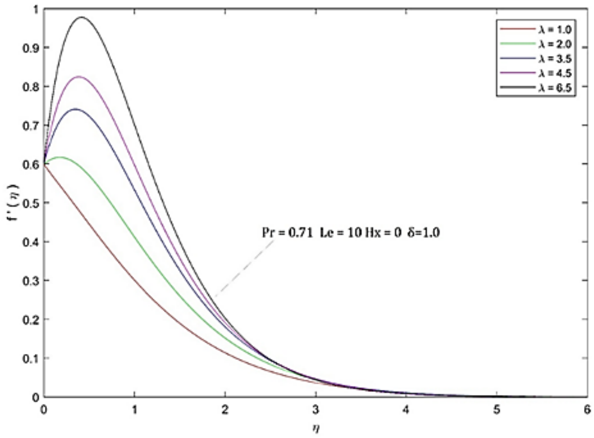


Fig. 7(a) ***Dissimilar values of λ for finding the stream wise velocity f'(η)***

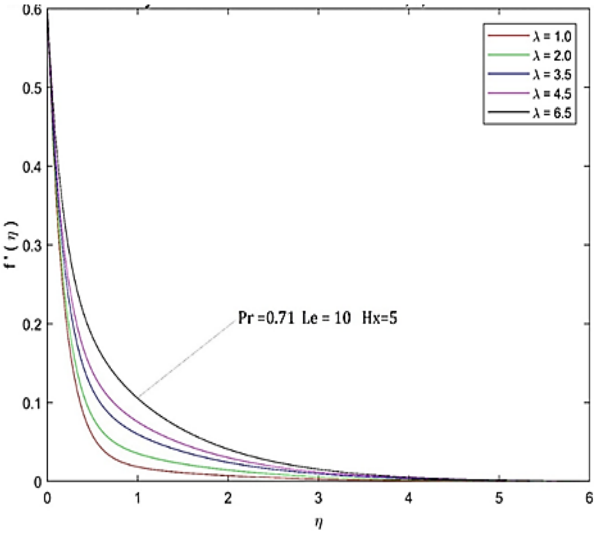


Fig. 7(b) ***Dissimilar values of λ for finding the stream wise velocity f'(η) with effect of Hartmann number***

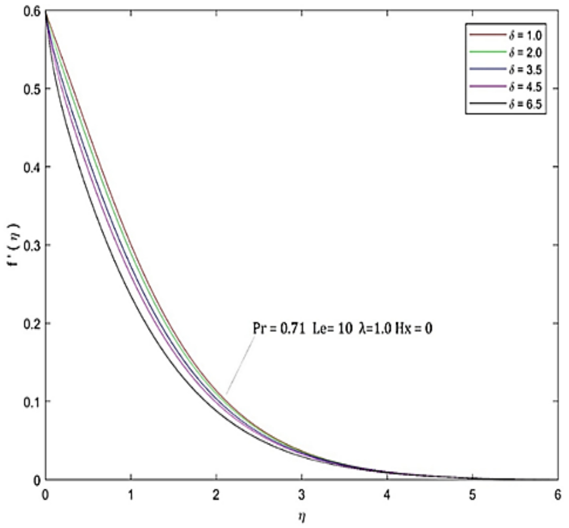


Fig. 8(a) ***Dissimilar values of δ for finding the stream wise velocity f'(η) with the effect of Hartmann number***

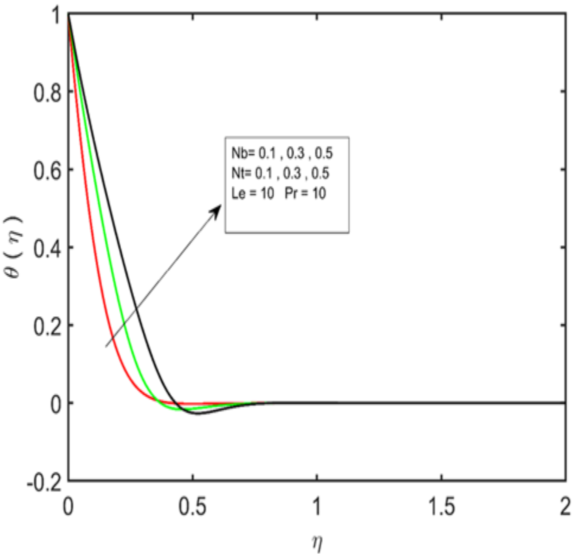


Fig. 8(b) ***Values of δ for finding the stream wise velocity f'(η) with the effect of Hartmann number***

The numerical configuration shows that in figure 8(a,b) present that taking constant values of Pr number and Le number the Velocity decreases when we increase the values of δ.

The effect of Hartmann number in figure 7(b) the velocity decreases because Lorentz forces attract toward the Nano particle the heat transfer decreases.

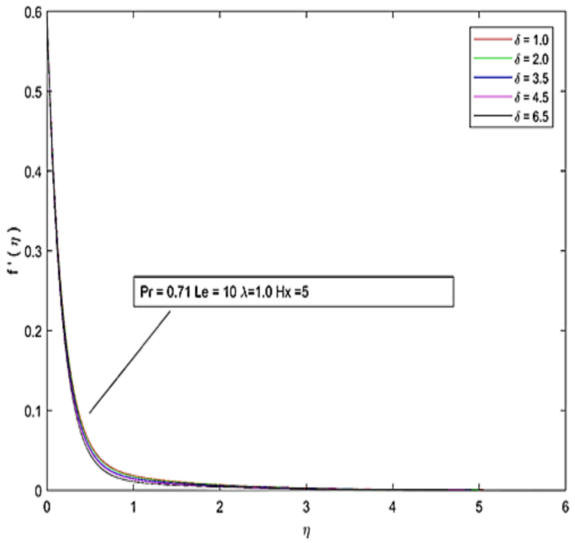


Fig. 9. ***Dissimilar values of δ for finding the stream wise velocity f'(η) with the effect of Hartmann number***

The effect of Brownian motion and thermophoresis parameters values increases the thermal boundary layer to be thicker which results in diffusion penetrating deeper into the fluid. In figure 9(a) when values Nb and Nt increases the temperature profile increases but under the effect of Hartmann number the maximum values of *Nb* and *Nt* affected directly it will be decreases because nano fluid particle temperature increases the strongest Lorentz forces create more enhancement and resolve sticky effect due to nanoparticle in the fluid that signified increase convective heat transfer.

Fig.10 when increases the Hartmann number then the temperature profile will rise upward on specified parameters.

Fig. 11 presents that increases Hartmann number the velocity will be decreased. The velocity profile/streamlines close to the hot plate.

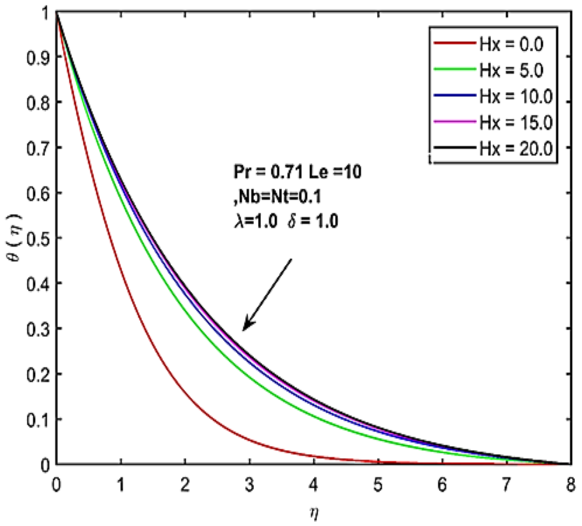


Fig. 10. ***Hartmann number effect on specified parameters for thermal profile***

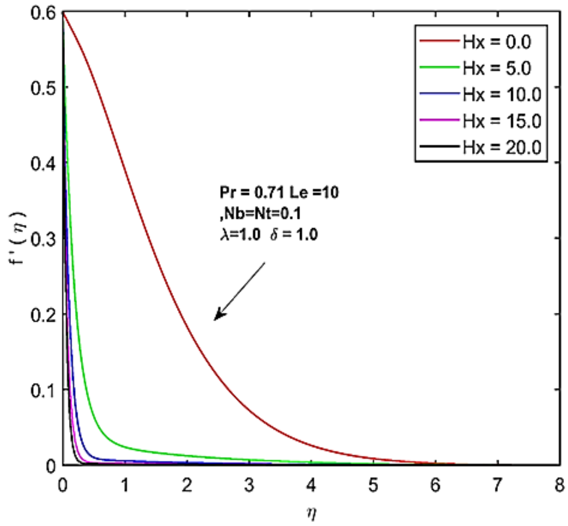


Fig. 11. ***Hartmann number effect on specified parameters for velocity profile***

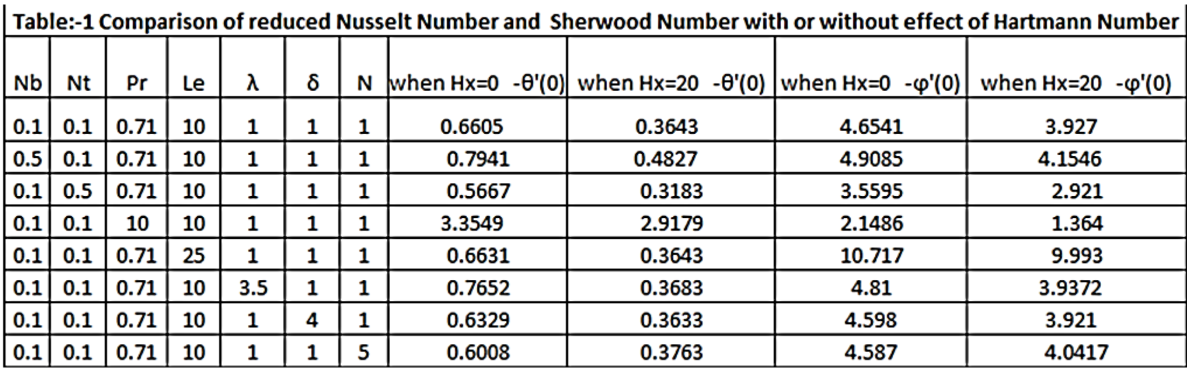


Table:-1 Comparison of reduced Nusselt Number and Sherwood Number with or without effect of Hartmann Number

Nb	Nt	Pr	Le	λ	δ	N	when Hx=0 -θ'(0)	when Hx=20 -θ'(0)	when Hx=0 -φ'(0)	when Hx=20 -φ'(0)
0.1	0.1	0.71	10	1	1	1	0.6605	0.3643	4.6541	3.927
0.5	0.1	0.71	10	1	1	1	0.7941	0.4827	4.9085	4.1546
0.1	0.5	0.71	10	1	1	1	0.5667	0.3183	3.5595	2.921
0.1	0.1	10	10	1	1	1	3.3549	2.9179	2.1486	1.364
0.1	0.1	0.71	25	1	1	1	0.6631	0.3643	10.717	9.993
0.1	0.1	0.71	10	3.5	1	1	0.7652	0.3683	4.81	3.9372
0.1	0.1	0.71	10	1	4	1	0.6329	0.3633	4.598	3.921
0.1	0.1	0.71	10	1	1	5	0.6008	0.3763	4.587	4.0417

The numerical result is described under the effect of the Hartmann number by using the buoyancy parameter and solute buoyancy parameter. The variation of reduced Nusselt number and reduced Sherwood number to take different values of the *Nb, Nt, Pr, Le, λ,* and δ.it is observed that $-\theta'(0)$ is increasing when we increase the values of *Pr, Le* and *λ* and decreasing when we increase values of *Nt*. The Hartmann number effect is shown in the table the overall values decrease with different range. The reduced Sherwood number decline for large values of Pr number because the fluid will be dense and Hx values affect the Pr number velocity decreases heat transfer increases.

5. Conclusion

In the present study, the nanofluids over a nonlinear horizontal stretched sheet investigated the conjugate heat and mass transfer. The velocity of Nano fluid variation with Prandtl number, solutal buoyancy parameter δ whereas increases for variation of buoyancy parameter λ Hartmann number decreases the velocity profile. Numerical solution depending upon all parameters describe through table and graph. Further following main results are concluded from this research.

1. When Hartman numbers a magnetic effect on the surface of the nonlinear stretching sheet then the thermal profile will be less due to hot plate magnetic intensity and the High value of Hx required for variation of thermal profile. When increases the Hartmann number the flow field increases due to the temperature rise of *Hx*. The temperature Profile of Nano fluids declines when Pr and λ are increased whereas increases for large value of *Pr, δ* and these high values of the *Pr*

number will represent the low thermal boundary layer affect the conduction phenomenon they thick the Nano fluid with high viscosity they generate the collide effect on the boundary layers. We detected that concentrated Nu number rises for the growing value of *Pr* whereas decreases for the increasing value of δ and *N*. but when Hartman number magnetic effect on it will be opposite.

2. The Concentration profile of the nonlinear stretching sheet decreases for large values of λ and increases for large values of *Pr,* δ and *N* if *Hx* effect on the surface of the stretching sheet then thermophoresis parameter and *N* effect on maximum. When increasing the *Le* number then the concentration profile decreases under the magnetic effect. The Hartmann number effect on concentration profile descends smoothly in the free stream when increasing the Hx.
3. We observe Nusselt number $-\theta(0)$ increases when *Pr* number increases but values of Hx direct effect on *Pr* values it will increase due to Lorentz forces and skin friction also increases for the greater value of *Pr*. The reduced Sherwood number $-\phi(0)$ in the table presents large values of *Le* number, *Nb* and λ decrease large values of *Pr* number and *Nt*. In the future, we used different types of Nano fluids such as *Cu*, Al_2O_3 and TiO_2 for analysis on stretching of the sheet for finding conjugate heat transfer.

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